Is positive income elasticity of demand really associated with normal goods?: revisiting the Slutsky equation and net benefit ratio

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Abstract

Incorporating a household’s net sale status into a rearranged Slutsky equation with combined ordinary and endowment income effects, this paper aims to reinterpret the income elasticity of demand in the case of buying and selling and to associate it with types of goods in a novel manner. To this end, the Deaton’s (1989) net benefit ratio (NBR) approach is expressed as the difference between original and endowment budget shares, and formulated in its elasticity form as the difference between the Hicksian and Marshallian own-price elasticities at any given price, divided by the income elasticity. While the numerator in the latter expression is always positive (negative) for normal (inferior) goods, the denominator may be either positive or negative for either type of good, depending on the net sale position of the household. A positive NBR for a normal good implies that the household is a net demander of that good and that the income elasticity is positive. When the NBR is negative for such a good, it implies that the household is a net seller and that the income elasticity is negative. Again, a positive NBR for an inferior good refers to the fact that the household is a net demander and the income elasticity is negative, whereas a negative NBR reveals that the household is a net seller of that good, which has, unconventionally, a positive income elasticity.

Keywords: NBR, Slutsky equation, household net sale, income elasticity

Introduction

In economics, consumers make decisions by allocating a given level of income to a bundle of commodities to maximize their utilities. Income elasticity of demand measures the sensitivity of the quantity demanded to a change in income, holding other things constant. As cited in the economics literatures (e.g. Frank, 2008 and Perloff, 2012), this elasticity is positive for a normal good and negative for an inferior good. However, in reality, the individuals sell what they initially own, i.e. their endowments,
to earn income and to make their purchase decisions (Varian, 2010). In such a case, income is determined by the value of endowment and rises with increases in prices of goods. This paper shows how, in such circumstances, the association of income elasticity with the type of good depends on the net sale status of consumers. In other words, this paper shows that the conventional relationships between income elasticity and type of good are true only if the person is a net buyer (demander) of the commodity in question. The relation is reversed when the consumer is a net seller (supplier) of the good. As an example, a positive income elasticity of demand (for instance) is associated with normal foods for urban consumers and landless rural households who are typically net food buyers, and negative for commercial farmers who are net food sellers and mainly live in rural areas.

Following Varian (2010), a household is a net seller (buyer) if net demand, i.e. the difference between gross supply, that is the value of endowment, and gross demand is positive (negative). If the price of a normal good increases for a net demander, then the household will buy less of it. Moreover, depending on the magnitude of the price change, because of the change in the value of endowment, the household may switch to become a net supplier.

Household net-seller/buyer status is conceptually explained in the literature (e.g. Minot & Goletti, 2000; Kytchnikova & Diop 2006; Benson et al. 2008; and the market analytical tool of the World Food Program, 2009). However, the net benefit ratio (NBR) introduced by Deaton (1989) has been widely used in many empirical works not only to determine the proportion of households who are net food buyers and sellers but to examine the impact of rising food prices and to assess the impacts of trade policies on household welfare (Budd, 1993; Barrett & Dorosh, 1996; Arndt et al., 2008; Ivanic & Martin, 2008; Pide and Kimsun, 2012; Van Campenhout et al., 2015). The conventional NBR is the difference between the consumption ratio (CR), the value of purchases and self-produced food, and the production ratio (PR), the value of sales and own consumption, both as a fraction of total household income. The NBR may be interpreted for a good as the before-response or impact elasticity of real income with respect to price movements in the good (Budd, 1993, p. 589; Minot & Goletti, 2000). It is negative for a net food supplier who would gain from a price increase, and positive for a net food demander who would lose.
The primary concern of this paper is to address the fact that a positive income elasticity is not associated only with a normal good nor is a negative income elasticity associated only with an inferior good. Moreover, the classification of household buying and selling decisions depends not only on the price of the good in question but also on the type of good, i.e. whether the good is normal or inferior. To this end, an alternative measure of the NBR in its elasticity form is derived by rearranging the Slutsky equation such that it depends on the difference between the Hicksian and Marshallian own-price elasticities and on the income elasticity of demand. This expression allows for including the magnitude of the combined income effect (ordinary income effect plus endowment income effect), and in particular reveals that the net sales position depends on the sign of income elasticity that, as is proved in this paper, may be positive or negative for both normal and inferior goods.

The rest of this paper is devoted to the methodology of deriving the elasticity form of the NBR and the extent to which it relates to the sign of income elasticity. The methodology is empirically examined at the end of the paper, by applying the demand and income elasticities for Iranian meat consumers.

**Elasticity form of the NBR**

Equation 1 is Slutsky equation in terms of rates of change (Varian, 2010, p170) in which the total effect of price change, \( \frac{\Delta x_i}{\Delta p_1} \), is decomposed into the substitution effect, \( \frac{\Delta x_i^s}{\Delta p_1} \), the ordinary income effect, \( -x_i \frac{\Delta x_i^m}{\Delta m} \), and the endowment income effect, \( \frac{w_i \Delta x_i^m}{\Delta m} \) (defined as the product of the change in demand when income, \( m \), changes, \( \frac{\Delta x_i^m}{\Delta m} \), and the change in income when \( p_1 \) changes, \( \frac{\Delta m}{\Delta p_1} \), that is, the endowment of \( x_i \)):

\[1\]

1 The “combined income effect” is a new microeconomics phrase introduced by Varian (2010, p. 169) to show that, considering the net sale status of consumer, total price effect has two income effect components, namely ordinary income effect and endowment income effect. The ordinary income effect refers to the change in quantity demanded that results from the change in real income due to the price change. In other words, it exhibits the change in quantity demanded holding the money income constant. The endowment income effect is an extra income effect that captures the changes in money income due to change in the price of the good. In other words, it reflects the influence of price change on the value of endowment bundle and thus on money income.
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\[
\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^m}{\Delta m} + W_1 \frac{\Delta X_1^m}{\Delta m}
\]

(1)

where \( x_j \) and \( p_j \) indicate respectively the quantity demanded and price of good 1, and \( \Delta x_1 \), \( \Delta x_1^m \) and \( \Delta x_1^s \) stand for total change in quantity demanded, and the changes in quantity demanded due to substitution and income effects, respectively.

Equation (1) in its elasticity form can be rewritten as in (2) below, where \( \epsilon_1 \) and \( \xi_1 \) are the Marshallian and Hicksian own-price elasticities, and \( \eta_1 \) is the income elasticity of demand for good 1:

\[
\frac{\Delta x_1 p_1}{\Delta p_1 x_1} = \frac{\Delta x_1^m p_1 m}{\Delta m x_1 m} + \frac{\Delta x_1^s p_1 m}{\Delta m x_1 m}
\]

(2)

\[
\epsilon_1 = \xi_1 - \frac{p_1}{m} \eta_1 (x_1 - w_2)
\]

(3)

\[
\epsilon_1 = \xi_1 - (\alpha_1 - \alpha_1^m) \eta_1
\]

(4)

where \( \alpha_1 - \alpha_1^m \) is the difference between the original and endowment budget shares of good 1. Hence, \( \frac{\Delta x_1}{\Delta m}(x_1 - w_2) \) is implicitly the NBR and thus can be expressed as:

\[
NBR = \frac{\Delta x_1}{\Delta m}(x_1 - w_2) = \alpha_1 - \alpha_1^m = \frac{1 - \epsilon_1}{\eta_1}
\]

(5)

Equation (5) shows that the NBR can be expressed as the difference between original and endowment budget shares, and formulated in its elasticity form as the difference between the Hicksian and Marshallian own-price elasticities at any given price, divided by the income elasticity.

Whether a household is a net buyer or net seller of good 1 can be determined by the NBR being positive and negative in its various forms in equation (5). However, a few remarks need to be considered in empirical works. First, \( \alpha_1^m \) is not directly measurable, although it can be approximated by the partial elasticity of income with respect to the price of the good, \( p_1 \), i.e. \( \epsilon_{mp} = \frac{\Delta m p_1}{\Delta p_1 m} \) and may be calculated as equation (6) when the change in income is due only to the (change in the) price of good 1:

\[\frac{\Delta m}{\Delta p_1} = \alpha_1 \frac{1 + \epsilon_1}{\eta_1}\]

where \( \epsilon_{j1} \) is cross demand elasticity of \( j \)th good wrt \( p_j \).
Substituting $\varepsilon_{mp}$ for $\varepsilon_1$ in equation (3) leads to the Slutsky equation appearing as

$$\varepsilon_1 = \xi_1 - (a_1 - \varepsilon_{mp})\gamma_1,$$

and net selling can also be judged by the term in brackets.

Second, equation (4) suggests that $\xi_1$ needs to be calculated as a residual taking into account the endowment budget share alongside the original budget share of the good, i.e. $\xi_1 = \varepsilon_1 - (a_1 - \varepsilon_{mp})\gamma_1$. Ignoring $\xi_1^m$, that is applying the classic version of Slutsky equation, leads to an incorrect measure of NBR and thus to miscalculation of the household net sale status.

The above discussion has shown that the NBR alongside the Slutsky equation provides a basis to know whether the household is a net demander/supplier of either a normal or inferior good. Worthy of note is the fact that the income elasticity of demand takes on a different interpretation by taking into account the combined income effect to formulate the elasticity form of NBR. This is discussed below in more detail.

**An alternative interpretation of income elasticity**

As stated earlier in the introduction, instead of a given income, a household is assumed to start off in reality with an endowment of goods. As such, the household original optimum bundle locates on the original budget line that crosses the endowment bundle. The budget line pivots inward or outward as results of increasing or decreasing the price of a given good, *ceteris paribus* (e.g. Varian, 2010). This line is called the pivoted budget line hereafter and could have been the final budget line in absence of the endowment income effect. However, because of the endowment income effect, the final budget line is parallel to the pivoted budget line and crosses the endowment bundle.

Applying a Slutsky decomposition of price change, i.e. drawing a new budget line through the original bundle that has the same price ratio as the pivoted budget line, breaks up the total price effect into substitution effect and two income effects. The movement from the optimal bundle on the new budget line to the optimal bundle on the pivoted budget line reflects the ordinary income effect and exhibits the change in quantity demanded holding the money income constant. The endowment income effect reflects the movement from the optimal bundle on the new budget line to
the final optimal bundle and captures the changes in money income due to change in the price of the good.

Ignoring the household net sale status, the household’s real income falls when the price of a good rises, *ceteris paribus*. So, for a normal (inferior) good, the income effect causes the quantity demanded to fall (rise) because of the positive (negative) income elasticity. In the case of household net sale, the income elasticity can still be attributed to the direction of the income effect, however, as discussed below through Figures 1-3, the relationship between change in real income and income elasticity is not so straightforward.

It should be noted that the ordinary (Marshallian) and income-compensated (Hicksian) demand curves respectively correspond to total effect, movement from the original bundle to the final bundle, and substitution effect, movement from the original bundle to that on the new budget line.

The difference between the Hicksian and Marshallian own-price elasticities (\( \xi_1, \varepsilon_1 \)) is always positive (negative) for a normal (inferior) good, no matter if the household is a net demander or net supplier of that good. However, the sign of the income elasticity (\( \eta_1 \)) depends on the type of the good as well as on the net sale position of the household. This argument is graphically shown in Figure 1. In both Figures 1a and 1b, bundles A and B represent the original and the after price change optimal choices for a net buyer household.

Depending on the degree of price change, a buyer household may or may not switch to being a net seller after the price change. Regardless of the net sale position of the household and whether \( p_1 \) decreases or increases, the substitution effect is always negative, as represented by the movement from A to C. It is assumed in Figure 1 that the household is a net buyer and stays a net buyer after increasing \( p_1 \), and so the final choice B as well as the bundle C are located somewhere to the right of the endowment on the original budget line. However, the positions of A, B and C differ for normal and inferior goods as shown in Figures 1a and 1b.
Applying the Slutsky decomposition to breaking up the total price effect into substitution and income effects results in point C, which indicates the case of a normal good for case $a$ and of an inferior good for case $b$. Referring to the top figure, the combined income effect, movement from C to B, causes the quantity demanded of normal (inferior) good to fall (rise). This implicitly implies that income elasticity ($\eta$), is positive for normal good and negative for inferior good and that for a net buyer household has its usual sign, positive for a normal good and negative for an inferior good.

Thus, including endowment does not make any difference to the classic conclusion of consumer choice, and so the difference between Hicksian and Marshallian elasticities ($\xi_1 - \varepsilon_1$) at any given price is respectively positive and negative for normal and inferior goods, as shown in the lower parts of Figure 1. This is because of the fact that, regardless of net selling status of household, the compensated demand curve, $D_h$, crosses the ordinary demand curve, $D_m$, from above for a normal good and from underneath for an inferior good.

When a household is a net seller of the good, the income elasticity, $\eta_1$, is negative for a normal good and positive for an inferior good, as shown in Figure 2, and discussed below. Worthy of note is that the final bundle is on the solid bold line, i.e. the final budget line, not on the pivoted budget line.
line as is the case in conventional decomposition of total price effect. This is critical in the framework of this paper because the movement from C to any point on these two lines respectively implies to a decrease and an increase in household income and thus to different signs of income effects which in turn ends up to either negative or positive income elasticity.

Here, the household is a net seller of $X_1$ before the price change, where A is the original bundle. Because of the endowment income effect, the choice is B after the price of $X_1$ drops. Applying the Slutsky approach to determine substitution and income effects ends up at C, that is somewhere between A and B for a normal good and to the right of point B for an inferior good. As happens in Figure 2a, the bundle C (or even B) may be at the same level of good 1 as that of the endowment W. As always, movement from A to C represents the substitution effect and is negative for both cases 2a and 2b.

Figure 2. Substitution and income effects of price decrease for a switching net seller

The final choice B locates on the bold budget line passing through the endowment bundle W. In case of a decrease in the price of $X_1$ shown in Figure 2, the final income or the endowment value is less than the original income, whereas the combined income effect, the movement from C to B, is negative for a normal good and positive for an inferior good. This implies that, when the household is a net seller of the good, the income elasticity is unexpectedly negative for the former but positive for the latter type of good.
Similar to the case of net buying, $D_h$ crosses $D_m$ from above (underneath) for a normal (inferior) good and thus $\xi_1 - \epsilon_1$ is again positive (negative) for a normal (inferior) good at any given price.

A closer look at the above discussion is shown in Figure 3. When the price of normal good $X_1$ increases, the original budget line (solid line 1) pivots to the solid line 2, but, because of the endowment income effect, the final budget line passes through the endowment bundle as shown by the bold solid line 3. The original bundle is $A_1$ if the household is a net demander of $X_1$ and $A_2$ if she is a net supplier of that good. After the price change, the net buyer chooses bundle $B_1$, assuming that she switches to being a net supplier, and the net seller chooses the bundle $B_2$, both of which are located somewhere to the left of their original choice on the bold solid line 3.

![Figure 3. Substitution and income effects for net buyer (seller) of a normal good](image)

Applying the Slutsky approach to decomposing total effect, that is drawing the two dotted lines, ends up with $C_1$ and $C_2$ respectively for the net buyer and net seller. The substitution effect in both cases is negative (movement from $A_1$ to $C_1$ and from $A_2$ to $C_2$). However, the combined income effect of the price increase has not the same sign for the two net sale positions. Whilst income decreases by moving from $C_1$ to $B_1$, it increases by moving from $C_2$ to $B_2$; however, the respective quantities decline in both cases. In other words, the income effect, and hence the income elasticity,
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is negative for a net seller but positive for a net buyer of the normal good even if she switches to being a net seller after the price increase.

Keeping this in mind and referring to equation 5, it is now straightforward to judge the net sale position of household based on the demand elasticities $\eta_1$, $\varepsilon_1$ and $\xi_1$ as summarized in Table 1:

<table>
<thead>
<tr>
<th>Type of good</th>
<th>$\xi_1-\varepsilon_1$</th>
<th>$\eta_1$</th>
<th>NBR</th>
<th>Net buying position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>Buyer</td>
</tr>
<tr>
<td></td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>Seller</td>
</tr>
<tr>
<td>Inferior</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>Buyer</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>Seller</td>
</tr>
</tbody>
</table>

In practice, when a consumer is a net seller (net buyer) of a normal or inferior good, she regards the market price as so high (low) that she would prefer to consume less (more) than she initially owns. If the price goes up (down), she wants to remain a net seller (buyer), and is expected to consume even less (more) than before. As far as the substitution effect is concerned, her response to a price change is to change her consumption in the reverse direction. However, an increase (decrease) in price of a good implies that the income of a net seller of that good rises (decreases), as can be seen by comparing the bold and dotted budget lines in upper parts of Figure 2 for the case of decreasing price. The point here is that the income effect of a normal (inferior) good is unexpectedly negative (positive) for the net seller of the good. In other words, the income elasticity is negative (positive) for a normal (inferior) good if the household is originally a net seller (buyer) of that good.

Empirical findings

The estimated coefficients and crude meats and fish demand data were taken from Layani and Bakhshoodeh\(^1\) (2016) to calculate corresponding elasticities and the NBR separately for 17726 Iranian urban households in

\(^1\) In this study, the Deaton and Muellbauer (1980) linear approximation of the Almost Ideal Demand System (AIDS) was applied to Iranian meat and fish consumers, imposing demand properties including homogeneity, symmetric and adding-up, and applying the two-step procedure of Shonkwiler and Yen (1999) to account for zero consumption.
The ordinary own price and income elasticities ($\varepsilon_1$ and $\eta_1$) were directly calculated by the Almost Ideal Demand System formulae and the Hicks elasticity coefficients were calculated as $\xi_1 = \varepsilon_1 + (\alpha_1 - \varepsilon_{mg})\eta_1$.

Finally, the type of meats and fish (normal or inferior) and the net sale status of individuals were realized by the rules given in Table 1. Based on the calculated elasticities for all households and applying equation (5), the number and proportion of net demanders (suppliers) of meats and fish were determined and are shown in Table 2. As can be seen, a substantial number of households have zero consumption of meats and fish. Excluding these households, the majority of the households were net buyers of red and white meats, which are staple foods among the households in this country.

### Table 2. Distribution of meat and fish consumers by type of good and net sale status

<table>
<thead>
<tr>
<th></th>
<th>Valid cases</th>
<th>Zero consumption</th>
<th>Normal (buyer)</th>
<th>Normal (seller)</th>
<th>Inferior (buyer)</th>
<th>Inferior (seller)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>14066</td>
<td>3660 (20.6)</td>
<td>12680 (71.5)</td>
<td>80 (0.45)</td>
<td>0 (0.0)</td>
<td>1306 (7.4)</td>
</tr>
<tr>
<td>White</td>
<td>16478</td>
<td>1248 (7.0)</td>
<td>12118 (68.4)</td>
<td>490 (2.8)</td>
<td>0 (0.0)</td>
<td>3870 (21.8)</td>
</tr>
<tr>
<td>Fish</td>
<td>8412</td>
<td>9314 (52.5)</td>
<td>4326 (24.4)</td>
<td>96 (0.54)</td>
<td>0 (0.0)</td>
<td>3990 (22.5)</td>
</tr>
</tbody>
</table>

Percentages in brackets

Out of the total 17726 households, 71.5% are net buyers of red meat as a normal good. Red meat is also a normal good for 80 net seller households, for whom the income elasticity is negative. For the other households who are net sellers of this meat, red meat is an inferior good. For white meat, 68% of all consumers are recognized to be net buyers. Moreover, white meat is a normal good for 490 households (2.8%) who are net sellers and have a negative income elasticity.

The consumption of fish (as well as other sea foods) is relatively rare in Iran, and, as indicated, more than half of the families have zero fish consumption (9314 out of 17726 households). However, over 24% of the population are net fish buyers and recognize it as a normal good. Fish is inferior for 22.5% of total sample who are net sellers.

Referring to Table 2, the sign of income elasticity is unconventional for net sellers of either normal or inferior goods. According to the empirical
findings of this study, the vast majority of net sellers are those who recognize meat and fish as inferior goods and so it may be concluded that such goods stand to have unconventional sign of income elasticity of demand much severer than the normal goods.

**Concluding remarks**

Based on the analysis of this study, a positive or negative income elasticity of demand may be associated with both normal and inferior goods. If the household is a net buyer, the sign of this elasticity is positive for a normal good and negative for an inferior good. However, this is reversed when the household is a net seller of the good in question. As discussed, this derives from the fact that the income of a net seller household of a normal good (for instance) rises with an increase in price of that good, while, as far as the combined income effect is concerned, the consumption of that good may change in the reverse direction. A similar discussion implies that the income elasticity is unconventionally positive for a net seller of an inferior good because of the direction of its income effect. Thus, we do not know for certain whether a positive (negative) income elasticity is associated with a normal or inferior good unless we take the household net sales position into account.

Loosely speaking, it may be concluded from the findings of this study that households tend to be net buyers (sellers) of normal (inferior) goods; in other words, they want to consume more (less) normal (inferior) goods than they own. To sum up, net sales decisions made by individuals and households depend not only on the prices but also on the type of good in question.

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