Inflation Bias, Time Inconsistency of Monetary and Fiscal Policies and Institutional Quality

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Abstract:
In developing countries, weak institutional quality can increase the probability of applying discretionary policies and can have a great impact on their double-digit inflation. Surico (2008) calculated inflation bias, but he considered just monetary policy and he did not pay attention to the institutions. Therefore, we design a model which considers the discretion in monetary and fiscal policies and the effect of the institutional quality. Then we calculate the inflation bias resulting from time inconsistency of monetary and fiscal policies by solving our model. In fact, we used a Barro- Gordon type model for our purpose. After solving the model, sensitivity analysis is done. The experimental results of the model, for Iran’s economy as a developing country, show high degree of inflation bias during 1991-2016, furthermore, the weak institutional quality have positive effects on inflation bias.

JEL: E61, E62, E52, E58, H11

Keywords: Time Inconsistency, Institutional Quality, Inflation Bias.

1- Introduction
The high rate of inflation in many developing countries with weak institutional quality raises this question that: “Why the monetary authorities in these countries are not able to control inflation?” Although most of explanations about inflation have been presented based on the analysis of time inconsistency4 of Kydland and Prescott (1977) and Calvo (1978), few researchers have addressed this issue in studies about developing countries. The common aspects of almost all of these countries are to manipulate discretionary monetary and fiscal policies. Weak institutional quality of these economies reinforces the probability of applying these policies.

1. Strotz (1955) said that time inconsistency is because of changing preferences during the time. This definition has not use a lot. Johnes and Mahajan (2015) studies time inconsistency based on this definition.
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Institutional quality can be the cause of applying discretionary policy and may also contain inconsistency solutions. In fact, the proposed solutions for time inconsistency have entranced the discussion of institutions into time inconsistency literature. Among these solutions, Rogoff conservative model (1983) has had remarkable application in time inconsistency studies. Huang and Wei (2006) have demonstrated that in developing countries with low institutional quality, the central bank should be less conservative.

Surico (2008), obtained inflation bias equals with the difference between the optimal inflations in discretion and commitment situation. But he considered just monetary policy. Besides, he did not pay attention to the institutions.

Since in developing countries, the central bank is not independent, the issue of delegation of the bank’s authority to a conservative person does not matter. Therefore, one contribution of this study is that we have tried to make use of good governance indicators to clarify the role of institutions in time inconsistency. Thus, among the indicators of good governance, the two indicators of government effectiveness and regulatory quality have been introduced in the model.

In the end, by using data from Iran’s economy as a developing country, the model is calibrated and the average of inflation bias during 1991-2016 has been calculated.

The paper is organized as follows. Section 2 is devoted to theoretical considerations. Section 3, sets up the model. Section 4, drives the inflation bias via two approaches of discretion and commitment. In the last part of section 4, sensitivity analysis is conducted. Section 5, draws conclusions.

2- Some Theoretical Considerations

The models used for the analysis of time inconsistency of monetary and fiscal policies (simultaneously) are categorized into two groups:
- Lucas and Stokey-type Models
- Barro and Gordon-type Models

Lucas and Stokey (1983) designed a model for analyzing time inconsistency of monetary and fiscal policies. They used the Ramsey (1927)’s model. Because of its equilibrium condition and special structure, Lucas and Stokey’s model has usually been considered as the basis for analyzing the multiple equilibria or expectations trap. Some studies such
as Himmels and Kirsanova (2013) and Bai and Kirsanova (2014) used this model.

While, Barro and Gordon (1983) designed a model consist of the central bank loss function and Philips curve as the optimization constraint and examined the discretionary and commitment behavior of the central bank. After them, many researchers have tried to augment their model by adding the fiscal policy and considering the monetary and fiscal interactions. The first augmented model was proposed by Alsina-Tabellini (1987) has become the basis for many of simultaneous time inconsistency of monetary and fiscal policies.

The models based on Barro-Gordon are two groups. The first one, by adding the fiscal policy-maker, has considered a distinct loss function for him. For instance, Dixit and Lambertini (2000, 2003). The second group has regarded a general loss function for the government and the central bank so that in them, the inflation goal and the desirable output level is the same for both policy-makers. In such groups of models, the variables of fiscal policy such as government expenditures, tax and budget deficit are also embedded in loss function. Besides, the Philips curve is also generalized by adding the government’s fiscal instruments (e.g., Huang and Wei, 2006; and Bohn, 2009).

But in all these models, despite the presence of fiscal policy, the main part of the discussion has still been the monetary policy and it is assumed that the fiscal policy has always been applied in a discretionary way so they do not consider commitment condition for fiscal policy. Thus, it seems that this issue can be considered as a great weakness for this group of models of Barro-Gordon type.

Therefore, this paper will apply Barro-Gordon type approach to calculate the inflation bias of time inconsistency of the monetary and fiscal policies. The contribution of this paper is that by adding new assumptions, one is able to observe the commitment to both monetary and fiscal policy makers.

2-1- Institutional Quality and Time Inconsistency

Researchers have proposed various methods for solving the problem of time inconsistency. Since in time inconsistency of monetary policy, the assumption is that the central bank, to set the inflation rate, make the marginal cost of inflation equivalent with its marginal benefit, most of the solutions change the basic framework in a way that the marginal cost of
inflation that impose on the central bank will be risen so that incentive of the central bank decreases for inflation\(^1\).

The noteworthy point is that these proposed solutions have entranced the discussion of institutional quality to the inconsistency literature.

Among the solutions, the model of authority delegation proposed by Rogoff (1983), has been regarded as the most famous and the most widely used model in the literature of inconsistency and is known more by the title of “Rogoff Conservative Model”.

The discussion of institutional quality can be proposed in the form of “Good Governance.”

The indices of good governance include voice and accountability, political stability and absence of violence, government effectiveness, and regulatory quality, the rule of law and control of corruption. In this study, we have tried to use two indicators of effectiveness of government and quality of regulation in time inconsistency model.

3- The Model

In the present study, the framework of Barro-Gordon has been applied to calculate the inflation bias of time inconsistency of monetary and fiscal policies. These models consist of two main sections: loss function and the supply curve.

According to the structure of the economy of the developing countries, the fiscal and monetary policies are dependent and are determined by a third person (the president). Thus, a general loss function which consists of fiscal and monetary variables is used.

In this case, it is possible to follow Merzlyakov (2012), to add the bargaining parameter of both monetary and fiscal policy to the general loss function. In fact, bargaining parameter of the relative fiscal to monetary policy can show the capability of the government in formulating and

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\(^1\) The most important of these solutions are as follow:

- **Binding rules**: a monetary policy based on rules rather than discretion.
- **Delegation**: Based on this solution, the monetary policy-making is granted to someone who avoids inflation. In this solution, it is assumed that the preferences of the central banker about inflation are different from the society. In fact, the central banker gives more care to reducing the inflation. Thus, the marginal cost of inflation which is imposed on the central bank will be more than the people.
- **Reputation**: In this solution, the reputation and credibility enters into a repeated game so that the motive for inflation destroys the credibility of the central bank for reaching low inflation. Consequently, people expect higher inflation in the future. Reputation loss by punishing from the central bank increases the marginal cost of inflation.
- **Incentive contracts**: these are the arrangements in which the central bank is punished (either financially or through loss of credibility).
implementing the economic policies. That is why it has been considered as an indicator of institutional quality (quality of government regulation). The more value of this parameter increases the weight of the fiscal policy relative to the monetary policy in policy-making and fiscal policy has more domination over monetary policy. With the increase in the fiscal domination, the probability of applying discretionary behavior for compensating the budget deficit and debt payment will increase and as a result more inflation bias can be observed.

Therefore, the general loss function including the variables of inflation, output gap and budget balance. In sum, the general loss function is written as Eq. (1):

$$L = \frac{1}{2} \left[ (1 + \omega) \left( \pi_t - \pi^*_t \right)^2 + \left( \alpha_{ym} + \omega \alpha_{sf} \right) \left( y^* - k \right)^2 + \omega \alpha_{sf} x^2 \right]$$

(1)

Where, $x_t$, $\pi_t$, and $y^*$ are the ratio of budget deficit/surplus to the output, inflation rate and output gap, respectively. $\pi^*_t$, is the inflation target and $\omega$ is the bargaining parameter of both policies. $\alpha_{ym}, \alpha_{sf}$, characterize the priorities of central bank and government in output stabilization, respectively. $\alpha_{sf}$ is the weight that the government attaches to the budget balance. The parameter of $k > 0$ shows the monopolistic competition or the labor market distortions. In fact, due to the presence of these distortions, the output is inefficiently low.

Another interpretation for this parameter is related to the institutions. In other words, it is stated that presence of $k$ is due to the political pressures. In fact, the elected officials constantly tend to implement expansionary policies in order to get the people’s attention and to increase the chances of their re-election. Therefore, by reforming the institutions, the political pressures can be minimized (Walsh, 2010, p.272). Accordingly, policy-makers try to stabilize the output and inflation. The reverse of the parameter $k$ can also be used as the indicator of the effectiveness of the government; one of the other indicators of good governance. The indicator of government effectiveness, beside the quality of public and private services, involves also their amount of independence from political pressures. The more the amount of this parameter, the more the government’s activities affected by political pressures and the probability of applying discretionary policies will be increased and finally, the
inflation bias will be more. The smaller amount of $k$, the government is more independent from the political pressures and consequently, it has greater effectiveness.

The central bank, by setting the appropriate level of money supply or interest rates, can indirectly choose inflation as a policy-making tool. The goal of the central bank and government is to achieve a desirable level of inflation, but this issue has its own importance and weight for every policy-maker. The variable of tax or government expenditures can be employed as the fiscal instrument. In this paper, without accurate determination of this issue, it is assumed that the variable $x_t$ is the fiscal instrument. In other words, the fiscal policy-maker is able to use both of two instruments of (expenditures tools or tax) in order to create budget balance.

In Barro and Gordon- type models, the expectations-augmented Philips curve has been used for introducing the supply- side of economy\(^1\):

$$y^* = \beta_1(\pi_t - \pi_t^e) - \beta_2 x_t - u_t \tag{2}$$

Where $y^*, \pi_t, \pi_t^e$, are output gap, inflation and expectation forms during the period $t - 1$ about inflation in period $t$, respectively. The parameter $\beta_1$ represents the marginal benefit (extra output) that generate by the surprise inflation. The parameter $\beta_2$, displays the effect of fiscal policies on the output gap (the marginal benefit resulting from applying the expansionary fiscal policy) and since $x_t = T_t - G_t$ ($G_t, T_t$ are the government expenditures and tax, respectively), the negative sign represents the positive effect of expansionary policy on the output gap. It is assumed that, $u_t$ is the disturbance term in supply function that follows the first order autoregressive process ($u_t = \rho u_{t-1} + \epsilon_t$) and the private sector has rational expectations:

$$\pi_t^e = E_{t-1}\pi_t \tag{3}$$

Where $E_{t-1}$, is the conditional expectation on the information in the time $t - 1$.

\(^1\) In fact, Floden (1996), by comparing the different supply curves in these models has shown that the obtained results are not significantly different from each other
4- Solving the Model

The model solution will be done by two approaches of discretion and commitment. After applying the optimization, inflation bias will be the difference of results of these two approaches. Finally, the sensitivity analysis of the parameters has also been done.

4-1- Discretion

In the discretion approach, it is assumed that the government does not follow a specific rule in using its monetary and fiscal instruments and use them depending on the conditions. In this approach, the policy-maker has no control over the inflation expectations and optimization is just done in relation to the inflation and the fiscal variable. Thus, we minimize the general loss function (Eq. (1)) subject to Eq. (2). The results of the first-order condition are calculated and by combining the results, inflation due to discretion is obtained as Eq. (4):

$$\pi^D = \frac{\pi^e \left(1 + \mu \frac{(1 + \omega)\beta_k^2}{\omega \alpha_{z,1} \beta_1} \right) + \mu \left(\beta_1 \pi^e + u_t + k \right)}{1 + \mu \left(\frac{(1 + \omega)\beta_k^2}{\omega \alpha_{z,1} \beta_1} + \beta_1 \right)}$$ (4)

4-2- Commitment

By assuming that $z$ is the economic shocks, the two fiscal and monetary instruments are considered as $x(z), m(z)$. Minimizing the general loss function will be done by considering monetary and fiscal rules. About the inflation, it is assumed that:

$$\pi_t = \pi^e + au_t$$ (5)

Where $\pi^e$ is the expected inflation value and $a$ is the coefficient which shows how the supply shocks affect the inflation. About the fiscal instrument, it is also assumed that:

$$x_t = x_0 + bu_t$$ (6)

Where $x_0$ is the initial budget balance that the government is responsible to maintain so that in every period, the balance budget of $x_t$ just changes

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1. The results of the first and second order condition are presented in Appendix A.
proportionate to $b$ due to the occurrence of economic shocks. By this rule, before the formation of people’s expectations and before observing the shocks $u_t$, the central bank is committed to certain values of $\pi^e$ and the government is committed to certain values of $x_0$. After the occurrence of shocks, the policy-maker tries to choose $a$ and $b$ in a way that the expected value of the loss functions reaches its minimum amount. After differentiation from Eq. (1) subject to Eq. (2), in relation to $a$ and $b$, optimized inflation by the rule is represented as Eq. (7):

$$\pi^e = \pi^e + \frac{\mu u_t}{1 + \mu \left(1 + \omega \right) \frac{\beta_2}{\omega \alpha_{st} \beta_1} + \beta_1}$$  \hspace{1cm} (7)$$

4-3- Calculating Inflation Bias

In order to calculate the amount of inflation bias resulting from discretionary policies, the difference of inflation by two approaches of discretion (Eq. (4)) and commitment (Eq. (7)) is calculated. This difference has been shown as Eq. (8):

$$\pi^D - \pi^C = \frac{(\pi^* - \pi^e) \left(1 + \mu \left(1 + \omega \right) \frac{\beta_2}{\omega \alpha_{st} \beta_1} \right) + \mu k}{1 + \mu \left(1 + \omega \right) \frac{\beta_2}{\omega \alpha_{st} \beta_1} + \beta_1}$$  \hspace{1cm} (8)$$

Eq. (8), Shows that if the expected inflation ($\pi^e$) is lower than the target inflation ($\pi^*$), the inflation bias of applying discretionary monetary and fiscal policies will be more. In other words, if the people’s inflation expectations become lower than the inflation target, the incentive of the policy-maker to trade–off output with inflation and applying the discretionary policies will be more. Thus Eq. (8) confirms theory of Kydland and Prescott (1977) and that of Barro-Gordon (1983).

In addition, with the increase in the political pressures on the government ($k$) and in other words, with the decline of amount of its effectiveness, the inflation bias resulting from the discretionary policies increases. Because by increasing the political pressures, the policy-maker

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1 The results of the first and the second condition are provided in Appendix B.
pays more attention to the expansion of the output and his incentive increases for exchanging the output with inflation. Thus, the inflation bias resulting from the discretionale policies also increases. On the other hand, Eq. (8) is linked with the equation which Walsh (2010) has obtained for inflation by the monetary discretionary policy (without embedding the fiscal policy in the model).

Before calculating the inflation bias, the parameters $\beta_1, \beta_2, \mu, k, \omega, \alpha_{sf}, \alpha_{ym}, \alpha_{sf}$, the mean value of the expected inflation and target inflation should be calculated. For calculating two parameters of $\beta_1, \beta_2$, Eq. (3) has been used and the values of these two parameters are $0.0003$ and $2.5 \times 10^{-5}$ respectively. Then using the values of $\beta_1, \beta_2$ in the Eq. (A.2) and (B.5), the values $\mu = 80.48, k = .25$, have been calculated. The values related to $\alpha_{sf} = .75, \alpha_{ym} = 0.75, \alpha_{sf} = 1.5$, are also taken from the study of Merzlyakov (2012). In the following, by substituting the value of the parameters of Eq. (A.7), the value of parameter $\omega = 0.002$ has been extracted. For measuring the inflation bias, one of the parameters has been the target inflation. To calculate this parameter, mean values of specified target inflations in five development plans of Iran have been used, that is $11.82$. Inflation with one lag plus Exchange rate changes has also been considered as expected inflation whose mean value is $20.12\%$.

By replacing parameters values in Eq. (8), the amount of inflation bias obtained $9.8\%$. The observed inflation is $19.83$ but sum of inflation bias and target inflation is $21.62$ that are about $2\%$ more.

For the better result, we used Misery Index, instead of inflation. Misery Index is sum of inflation and unemployment, which is $30.82\%$ on average. By using Misery Index, policy bias is equal to $10.11\%$. If we add policy bias to target policy, the result is $30.88$ that is much close to observed Misery Index. Therefore, our results are improved.

4.4- Sensitivity Analysis

The results of the sensitivity analysis of the amount of inflation bias to some parameters of the model $\beta_2, \beta_1, \omega, \alpha_{sf}$ are shown in the following.

Fig. (1) Shows the analysis of the sensitivity of the inflation bias to the weighted coefficient of the budget balance. As it shown, the more the
government’s attention to maintain the budget balance, i.e. if $\alpha_{sf}$ increases, the inflation bias will rise. In other words, when the government pays more attention to compensating the budget deficit, if tax is not adequate to finance the government expenses, policy maker tries to compensate it by increasing using of oil incomes or borrowing from the central bank. All these cases will lead to increase monetary base and raise inflation.

![Graph](image-url)

**Fig (1). Sensitivity Analysis of Inflation Bias with Respect to the Balance Budget Coefficient**

**Source:** Research finding

Fig. (2) Shows the inflation bias sensitivity analysis to the bargaining coefficient, $\omega$. According to this figure, we can observe that increasing the bargaining power of the policy-maker, raises the inflation bias. Since the bargaining parameter of the policy-maker for choosing the fiscal policy relative to the monetary policy is the indicator of regulatory of the quality of government, this figure shows that if the bargaining parameter of fiscal policy relative to monetary policy increases, the index of regulatory the quality of the government will be lower. Because with increase in this parameter, the regulation of the policies will be towards fiscal policies and the central bank will become more passive.
In conditions of active fiscal policy and passive monetary policy, when taxes do not meet the government expenses, the seigniorage should establish government budget constraint. In these circumstances, the monetary policy must be adjusted so that sufficient seigniorage is made available to the government to meet the government’s budget deficit. Therefore, in accordance with the changes in prices and inflation, fiscal policies will change (Walsh, 2010, p.143). In sum, based on Fig. (2), it can be said that the weaker index of the quality of regulation, increases the probability of fiscal domination in the economy and raises the policymaker’s incentives to apply more discretionary policies. As a result, the inflation bias also increases.

Fig. (3) Shows the relationship between the marginal benefit of the inflation ($\beta_1$) with the inflation bias. In fact, if the increase of output due to surprise inflation become greater, ($\beta_1$ is greater), then the policy-maker is more inclined to use such discretionary policies. But based on model of Barro-Gordon (1983), this policy will lead to inflation. Thus, the policymakers in low levels of inflation compared to its high levels will be more motivated to trade-off output with inflation. Therefore, by increasing $\beta_1$, applying discretionary policies will increase and, consequently the inflation bias will increase proportionate to the amount of $\beta_1$. 

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**Fig (2). Sensitivity Analysis of Inflation Bias with Respect to the Bargaining Coefficient**

**Source:** Research finding
The amount of the changes of inflation bias based on the marginal benefit resulting from the application of expansionary fiscal policies $\beta_2$ is also shown in Fig. (4). Based on this, it can be observed that with the increase in the marginal benefit resulting from using the expansionary fiscal policy, the inflation bias will asymptotically move to zero (target inflation). In fact, as far as the policy-maker is able to increase the output using expansionary fiscal policies (increasing the expenses or decreasing the tax), he/she will pay less attention to the tradeoff between inflation and output and less incentive will be put into use surprise inflation. Then, the inflation resulting from using discretionary policies will be lower.
5- Concluding Remarks

Although economists have mentioned various determinants of inflation in developing countries with weak institutional quality, but the discussion of discretionary policies has less often attracted their attention. Therefore, in this paper we have tried to study a model of Barro-Gordon type, time inconsistency of monetary and fiscal policies at the same time. On the one hand, given that the institutional conditions in developing countries have certainly a great effect on the inflation bias, therefore, in this study with the use of a general loss function that has both monetary and fiscal policies at the same time, two indicators of good governance in the model, have thus been defined. The political pressures parameter has been used as the indicator of the government effectiveness and the bargaining parameter has been used as the indicator of regulation quality.

In fact, we have extended Walsh (2010)’s model by adding fiscal and institutional variables. So the results of these two models have been compared and showed that Walsh’s result can be obtained from our model by sacrificing adding parameters.

The experimental results of the model indicated high inflation bias in Iran’s economy (1991-2016) due to the presence of discretionary policies. Based on the result of the present paper, we suggest that:

First, central bank in Iran should be more independent. Second, government and central bank should avoid discretionary policies as much as they can.
References
Appendix A

The result of differentiation of the Eq. (1) subject to Eq. (2), relative to inflation, is obtained as Eq. (1):

\[(1+\omega)(\pi_i - \pi^*) + (\alpha_{ym} + \alpha_{sf})\beta_1(y^* - k) = 0\]  
(A.1)

\[\pi_i = \pi^* - \frac{(\alpha_{ym} + \omega\alpha_{sf})}{1 + \omega}\beta_1(y^* - k)\]  
(A.2)

For simplicity we put:

\[\frac{(\alpha_{ym} + \omega\alpha_{sf})}{1 + \omega}\beta_1 = \mu\]  
(A.3)

By substituting Eq. (3) and Eq. (A.3) in (A.2), we will have:

\[y^* = \pi^* - \mu \beta_1(\pi_i - \pi^*) - \beta_2 x_i - u_i - k\]  
(A.4)

And after simplification, one can obtain the optimized inflation in terms of the budget balance as the following equation:

\[\pi_i = \frac{\pi^* + \mu(\beta_1\pi^* + u_i + k)}{1 + \mu\beta_1} + \frac{\mu\beta_2}{1 + \mu\beta_1} x_i\]  
(A.5)

Also the results provided for the first-order condition with respect to the budget balance are obtained in the form of the following equation:

\[-(\alpha_{ym} + \alpha_{sf})\beta_2(y_i^* - k) + \omega\alpha_{sf} x_i = 0\]  
(A.6)

The results of the second-order condition of the public loss function of inflation and the budget balance are, respectively, shown in Eq. (A.7) and Eq. (A.8):

\[(1+\omega) + (\alpha_{ym} + \omega\alpha_{sf})\beta_1^2\]  
(A.7)

\[(\alpha_{ym} + \omega\alpha_{sf})\beta_2^2(y^* - k) + \omega\alpha_{sf}\]  
(A.8)

By assuming the parameters of the model as positive, it can be observed that both Eq. (A.7) and Eq. (A.8) are positive. It means that the
The general loss function has been concave with respect to inflation and budget balance function, and they have minimum value. By combining Eq. (A.6) and Eq. (A.2), one can obtain the optimized inflation with respect to the parameters of the model and those of the balance:

$$\pi_i = \pi^* - \frac{\omega \beta_i \alpha_{sf}}{1 + \omega} x_i$$  \hspace{1cm} (A.9)

By substituting (2) in (A.6) and making equivalent with \( \Phi \), we will obtain:

$$x_i = \frac{\Phi}{1 + \Phi \beta_2} (\beta_i \pi_t - \beta_i \pi^* - u_t - k)$$  \hspace{1cm} (A.10)

After substituting (A.10) in (A.5) we have:

$$\pi_i = \pi^* + \frac{\mu \beta_i \pi^* + u_t + k}{1 + \mu \beta_1} + \frac{\mu \beta_2 \Phi}{(1 + \mu \beta_1)(1 + \Phi \beta_2)} (\beta_i \pi_t - \beta_i \pi^* - u_t - k) = $$

$$\frac{\mu(\beta_i \pi^* + u_t + k)}{1 + \Phi \beta_2 + \mu \beta_1}$$  \hspace{1cm} (A.11)

Then, after substituting \( \Phi \) with respect to \( \mu \) as \( \Phi = \left( \frac{1 + \omega}{\omega \beta_i \alpha_{sf}} \right) \mu \), the inflation under discretion \( \pi^D \) will obtain as the Eq. (A.12):

$$\pi^D = \frac{\pi^* \left(1 + \mu \frac{(1 + \omega) \beta_2^2}{\omega \alpha_{sf} \beta_1} \right) + \mu \left( \beta_i \pi^* + u_t + k \right)}{1 + \mu \left( \frac{(1 + \omega) \beta_2^2}{\omega \alpha_{sf} \beta_1} + \beta_1 \right)}$$  \hspace{1cm} (A.12)
Appendix B:

The expectation value of Eq. (1) subject to Eqs. (5), (6) an. (2) can be written as (B.1):

\[
L = E_t \left[ \frac{1}{2} (1 + \omega) \pi_t^2 + au_t - \pi_t^2 + (\alpha_{ym} + \omega\alpha_{sf}) \beta_1 au_t - \beta_2 \left( x_0 + bu_t \right) - u_t \right]^2
\]

(B.1)

By differentiating from Eq. (B.1) with respect to \( a \), we can write:

\[
(1 + \omega) a \sigma_u^2 + E_t (\alpha_{ym} + \omega\alpha_{sf}) \beta_1^2 \sigma_u^2 - \beta_2 \beta_3 \beta_4 \sigma_u^2 = 0
\]

(B.2)

Where \( \sigma_u^2 \) is the supply shock variance and \( E_t \) is the expectation operator.

By simplifying Eq. (B.2), Eq. (B.3) is:

\[
a = \frac{\left( \alpha_{ym} + \omega\alpha_{sf} \right) \beta_1 (\beta_2 + 1)}{\left( 1 + \omega \right) + \left( \alpha_{ym} + \omega\alpha_{sf} \right) \beta_1^2}
\]

(B.3)

Besides, after differentiating Eq. (B.1) with respect to \( b \), we can write:

\[
b = \frac{\left( \alpha_{ym} + \omega\alpha_{sf} \right) \beta_1 (a \beta_2 - 1)}{\left( \left( \alpha_{ym} + \omega\alpha_{sf} \right) \beta_1^2 + \omega\alpha_{sf} \right)}
\]

(B.4)

The obtained value for \( b \) in the (B.4) is substituted in (B.3) and the new value of \( a \) is obtained as (B.5) below:

\[
a = \frac{\left( \alpha_{ym} + \omega\alpha_{sf} \right) \beta_1}{\left( 1 + \omega \right) + \left( \alpha_{ym} + \omega\alpha_{sf} \right) \beta_1^2 + \omega\alpha_{sf}} \beta_1 \left( 1 + \omega \right)
\]

(B.5)

Second-order differentiation of the general loss function or Eq. (B.1) with respect to \( a \) and \( b \) can be written, respectively, in Eq. (B.6) and Eq. (B.7):

\[
\sigma_u^2 \left[ 1 + \omega \right] + \left( \alpha_{ym} + \omega\alpha_{sf} \right) \beta_1^2 \beta_1^2
\]

(B.6)

\[
\sigma_u^2 \left[ \omega\alpha_{sf} + \left( \alpha_{ym} + \omega\alpha_{sf} \right) \beta_1^2 \right]
\]

(B.7)

By assuming that the parameters are positive, it can be observed that the general loss function relative to these two variables has been concave...
which minimum value. Then, the amount of inflation in conditions by the commitment of the fiscal and monetary policy-maker \((\pi^c)\) is Eq. (B.8):

\[
\pi^c = \pi^e + a_t = \pi^e + \frac{(\alpha_{m} + \omega \alpha_{y}) \omega \alpha_{y} \beta_1}{[(1+\omega)+\left(a_m + \omega \alpha_{y}\right)\beta_1]^{2}} \left\{ \left(a_m + \omega \alpha_{y}\right) \beta_2^{2}(1+\omega) \right\} u_t
\]  (B.8)

By utilizing Eq. (3-A), one can summarize the inflation by commitment as (B.9):

\[
\pi^c = \pi^e + \frac{\mu u_t}{1 + \mu \left( \frac{(1 + \omega) \beta_2^{2}}{\omega \alpha_{y} \beta_1} + \beta_1 \right)}
\]  (B.9)