International Journal of Business and Development Studies **Vol. 7, No. 1,** (2015) pp 31-50

The Comparison among ARIMA and hybrid ARIMA-GARCH Models in Forecasting the Exchange Rate of Iran

Mosayeb Pahlavani[*] and Reza Roshan[**]

Abstract

This paper attempts to compare the forecasting performance of the ARIMA model and hybrid ARMA-GARCH Models by using daily data of the Iran's exchange rate against the U.S. Dollar (IRR/USD) for the period of 20 March 2014 to 20 June 2015. The period of 20 March 2014 to 19 April 2015 was used to build the model while remaining data were used to do out of sample forecasting and check the forecasting ability of the model. All the data were collected from central bank of Iran. First of all, the stationary of the exchange rate series is examined using unit root test which showed the series as non stationary. To make the exchange rate series stationary, the exchange rates are transformed to exchange rate returns. By using Box-Jenkins method, the appropriate ARIMA model was obtained and for capturing volatilities of returns series, some hybrid models such as: ARIMA-GARCH, ARIMA-IGARCH, ARIMA-GJR and ARIMA-EGARCH have been estimated. The results indicate that in terms of the lowest RMSE, MAE and TIC criteria, the best model is ARIMA((7,2),(12)) –EGARCH(2,1). This model captures the volatility and leverage effect in the exchange rate returns and its forecasting performance is better than others.

Keywords: Forecasting Performance; Exchange Rate; ARIMA; GARCH Family Models; Volatility Modeling.

JEL classification: C45; C88; E37

1. Introduction

Forecasting the amount of economic variables by using appropriate models has always been very important to economists and policymakers. In other words, being aware of different forecasting models' abilities and identifying the most efficient model among the rival models is highly crucial in the process of policy making. For this reason, various models of estimating and forecasting economic variables have been created. One of the variables which play a basic role in international trade and finance for Iran's economics is exchange rate. Because, fluctuations in the exchange rate may have a significant impact on the

[*] Associate Professor, Department of Economics, University of Sistan and Baluchestan,, Iran((Corresponding author: email: pahlavani@eco.usb.ac.ir)

[**] Assistant Professor, Department of Economics ,Persian Gulf University, Bushehr, Iran

macroeconomic variables such as interest rates, prices, wages, unemployment, and the level of output. So, determination the behavior of exchange rate series is important for the investors and policy makers. Traditional economic models such as Box-Jenkins or ARIMA Model, assume that variance of residuals is constant, However in many cases residuals of the estimated model have heteroscedastic conditional variances. In such condition, one faces a stochastic variable with a heteroscedastic variance, and needs to forecast conditional variance or volatility of a time series. In this paper as later will be shown, the residuals of the estimated ARMA model for the exchange rate of Iran have conditional heteroscedasticity, hence it should be used models that are capable of dealing with the volatility of the exchange rate series. Therefore in this paper, for capturing volatilities of exchange rate returns, beside ARMA model, we use various volatility models such as Autoregressive Conditional Heteroskedasticity (ARCH), Generalized ARCH (GARCH, Integrated GARCH (IGARCH), Threshold GARCH (TGARCH) / the Glosten, Jagannathan and Runkle (GJR, Exponential GARCH (EGARCH. To evaluating the forecasting performance of various models, three different criteria have been used; consist of: Root Mean Squared Error (RMSE), Mean Absolute Error (MAE) and Theil Inequality Coefficient (TIC).

Following this Section 1, Section 2 reviews the existing literature and the empirical findings of the various models. Section 3 deals with the Methodology wherein formally defines theory and process of ARIMA, ARIMA-GARCH, ARIMA-IGARCH, ARIMA-JGR_GARCH and ARIMA-EGARCH models and introduced Performance Measures. The empirical results have been discussed in Section 4 followed by the conclusions which is given in Section 5. The reference can be found at the end.

2. Literature Review

A time series is a set of numbers that measures the status of some activity over time. It is the historical record of some activity, with measurements taken at equally spaced intervals with a consistency in the activity and the method of measurement.

The primary objective of time series modeling is to study techniques and measures for drawing inferences from past data. The models can be employed to describe and analyze the sample data, and make forecasts for the future. The main advantage of time series models is that they can handle any persistent patterns in data (Abdullah & Tayfur , 2004).

Accurate prediction of different exchange rates is important as substantial amount of trading takes place through the currency exchange market. The prediction is affected by economic and political factors and also involves uncertainty and nonlinearity. Thus accurate prediction of exchange rates is a complex task (Minakhi Rout et al, 2014). In the literature many interesting publications on exchange rate prediction have been reported as detailed in following.

Two broad ways can be applied for modeling and forecasting of exchange rate; one of them is multivariate approach that it is base on estimation relationship between exchange rate as dependent variable and some economic variable such as interest rate, output, money supply, inflation, balance of payment etc as explanatory variables. According to this, researchers and academics suggest a number of approaches to forecast exchange rate like; monetary approach, demand-supply approach, asset approach, portfolio balance approach and etc. Empirical studies use some of them very frequently especially monetary approach in different versions like flexible price monetary model (Frankel 1976 & Bilson 1978), the sticky price monetary model (Dornbusch 1976, Frankel 1979b) and Hooper–Morton model (Meese & Rogoff 1983, Alexander & Thomas 1987, Schinasi & Swami 1989 and Meese & Rose 1991). Franklin (1981) and Boothe and Glassman (1987) found that monetary/asset models are not very useful to explain the movements in exchange rates under flexible exchange rate system. John Faust et al (2002) examined the real-time forecasting performance of standard exchange rate models. A development in the focus came by the work of some of the researchers like (Taylor & Peel 2000; Taylor et al. 2001). They argued that underlying economic theories are fundamentally sound, still economic exchange rate models were not able to give superior forecasting performance because these models assume a linear relationship between the data. In reality these data shows nonlinearity. They argued that underlying fundamentals shows long run equilibrium condition only, towards which the economy adjusts in a nonlinear fashion (M.K. Newaz, 2008).

But this structural methodology has several limitations, which makes it less valuable in the field of finance. One such reason is that data for these macro economic variables are available at the most monthly, while in finance one need to deal with very high frequency data such as daily, hourly or even minutes wise also. Again, these structural models are not quite useful for out of sample forecasting. To avoid these problems, one often use univariate models or a-theoretical models which try to model and predict financial variables using information contained only in their own past values and possibly current and past values of an error term. One especial class of time series models are ARIMA models which are often associated with Box and Jenkins (1976) for their efforts to systematize the whole methodology of estimating, checking and forecasting using ARIMA models(Mahesh, 2005). The Box–Jenkins method consists of three steps: identification, parameter estimation and forecasting. Among these three steps, the identification step, which involves order determination of the AR and MA parts of ARMA model, is important. This step requires statistical information such as the autocorrelation and partial autocorrelation (Box and

Jenkins, 1976). The problem of estimating the order and the parameters of an ARMA model is still an active area of research (Rojasa et al., 2008). Building good ARIMA models generally requires more experience than commonly used statistical methods such as regression.

The Box-Jenkins variant of the ARMA model is predestinated for applications to non stationary time series that become stationary after their differencing.

Differencing is an operation by which a new time series is built by taking the successive differences of successive values, such as x(t) – x(t-1) along the non stationary time series pattern. In the acronym ARIMA, the letter "I" stands for integrated. The widely accepted convention for defining the structure of ARIMA models is ARIMA(p, q, d), where p stands for the number of autoregressive parameters, q is the number of moving-average parameters, and d is the number of differencing passes (Ajoy and Dobrivoje, 2005).

Bellgard and Goldschmidt (1999) predicted the exchange rates with the use of ARIMA models, However they concluded that these models are not very suitable for predicting the exchange rates. Dunis and Huang (2002) who were using ARMA (4,4) were of the opposite opinion; their results were, however, insignificant.

Weisang and Awazu (2008) presented three ARIMA models which used macroeconomic indicators to model the USD/EUR exchange rate. They discovered that over the time period from January 1994 to October 2007, the monthly USD/EUR exchange rate was best modeled by a linear relationship between its preceding three values and the current value. These authors also concluded that ARIMA (1,1,1) is the most suitable model for the prediction of the time series of USD/EUR exchange rate.

Fat Codruta Maria and Dezsi Eva(2011), using Box Jenkins models investigated the behavior of daily exchange rates of the Romanian Leu against Euro, United States Dollar, British Pound, Japanese Yen, Chinese Renminbi and the Russian Ruble using the exponential smoothing techniques and ARIMA models. The results indicate that exponential smoothing techniques in some cases outperform the ARIMA models.

Nwankwo Steve C(2014), applied Box-Jenkins methodology for ARIMA model to exchange rate (Naira to Dollar) within the periods 1982-2011 and it was proved test the best fit is AR(1) model, because it has the most suitable AIC. This was achieved through the diagnostic checking which identified it as the best fit.

Despite the fact that ARIMA is powerful and flexible in forecasting, however it is not able to handle the volatility and nonlinearity that are present in the data series. Previous studies showed that generalized autoregressive conditional heteroskedatic (GARCH) models are used in time series forecasting to handle volatility in the commodity data series including exchange rates. Hence, this study investigate the performance of hybridization of potential univariate time series specifically ARIMA models with the superior volatility model (GARCH family models), Combining models or hybrid the models can be an effective way to overcome the limitations of each components model as well as able to improve forecasting accuracy. In recent years, more hybrid forecasting models have been proposed applying Box-Jenkins models including an ARIMA model with GARCH to time series data in various fields for their good performance. Wang et al. (2005) proposed an ARMA-GARCH error model to capture the ARCH effect present in daily stream flow series. There is two-phase procedure in the proposed hybrid model of ARIMA and GARCH. In the first phase, the best of the ARIMA models is used to model the linear data of time series and the residual of this linear model will contain only the nonlinear data. In the second phase, the GARCH is used to model the nonlinear patterns of the residuals. This hybrid model which combines an ARIMA model with GARCH error components is applied to analyze the univariate series and to predict the values of approximation (S.R. Yaziz et al., 2013).

The Autoregressive Conditional Heteroscedasticity (ARCH) and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models were developed by Engle (1982) and extended by Baillie (2002) and Nelson (1991).

The first generation of GARCH models cannot capture the stylized fact that bad (good) news increase (decrease) volatility. This limitation has been overcome by the introduction of more flexible volatility specifications which allow positive and negative shocks to have a different impact on volatility. This more recent class of GARCH models includes the Exponential GARCH (EGARCH), the Glosten, Jagannathan, and Runkle- GARCH (GJR-GARCH) and the Power GARCH (PGARCH) model (Chatayan et al, 2010).

Some of the studies using hybrid model of ARIMA and GARCH family of models are as follows:

Balaban (2004) compared the forecasting performance of symmetric and asymmetric GARCH models with the US Dollar/Deutsche Mark returns series was filtered using an AR (1) process and the GARCH (1, 1), GJR-GARCH(1,1) and EGARCH(1,1) volatility equations are used. The author found that the EGARCH model performs better in producing out of sample forecasts with the GARCH (1, 1) closely following whereas the GJR-GARCH fares worst.

Moshiri and Seifi (2008), examined Nonlinearity in Exchange Rates of Iran and Forecasting it by ANN and GARCH models. The results show that ANN outperforms the GARCH model in forecasting the exchange rates, but generates the same results as the alternative models in forecasting the rate of change of the exchange rates.

Chatayan Wiphatthananananthakul and Songsak Sriboonchitta (2010), Compared among ARMA-GARCH, -EGARCH, -GJR, and -PGARCH models on Thailand Volatility Index (TVIX). The ARMA-PGARCH is found to be the best model with the lowest AIC criteria values but the ARMA-EGARCH model has the lowest SBIC criteria value. However, with the second moment condition, MAPE and RMSE, ARMA-GJR is the best fitting model for TVIX.

Shahla Ramzan et al (2012), applied ARCH family of models for modeling and forecasting exchange rate dynamics in Pakistan for the period ranging from July 1981 to May 2010 and ARMA(1,1)- GARCH (1,2) is found to be best to remove the persistence in volatility while ARMA(1,1)-EGARCH(1,2) successfully overcome the leverage effect in the exchange rate returns under study.

Milton Abdul Thorlie1 et al(2014), examined the accuracy and forecasting performance of volatility models for the Leones/USA dollars exchange rate return, including the ARMA, Generalized Autoregressive Conditional Heteroscedasticity (GARCH), and Asymmetric GARCH models with normal and non-normal (student's t and skewed Student t) distributions. Their findings showed that ARMA-GARCH) and ARMA-EGARCH model better fits under the non-normal distribution and the ARMA-GJR model using the skewed Student t-distribution is most successful and better forecast the Sierra Leone exchange rate volatility.

There are many studies that using various models for the modeling and forecasting foreign exchange rates data of USD versus Iran Rial (IRR); But so far, the hybrid ARMA and GARCH family models are not used for the modeling and forecasting IRR/USD. This paper offers insights on exchange rate in Iran and measures the sources of volatility by using Autoregressive Conditional Heteroscedasticity (ARCH), Integrated Generalized Autoregressive Conditional Heteroscedasticity (IGARCH), Exponential General Autoregressive Conditional Heteroscedasticity (EGARCH) and the Glosten, Jagannathan and Runkle (GJR-GARCH) techniques. Hence, the focus of presence paper is using the hybrid ARIMA-ARCH, ARIMA-IGARCH, ARIMA-EGARCH and ARIMA-GJR models for modeling and forecasting Iran's exchange rates that will be presented in the empirical study section.

3. Methodology

In this section, we briefly present the models specification, conditional distributions and forecasting criteria to model the volatility of Rial/US$ exchange rate returns in the Iran's economy. This article analyses the process and volatility of the Iran's exchange rate by using various models such as: AIRMA, AIRMA-GARCH, AIRMA-IGARCH, AIRMA-GJRGARCH and AIRMA-EGARCH. In this study three different criteria, Root Mean Squared Error (RMSE), Mean Absolute Error (MAE) and Theil Inequality Coefficient (TIC) are used to evaluate the forecasting performance of the various models.

3.1. The Box-Jenkins for ARIMA Model

Auto-Regressive Integrated Moving Average (ARIMA) model is one of the time series forecasting methods which says that the current value of a variable can be explained in terms of two factors; a combination of lagged values of the same variable and a combination of a constant term plus a moving average of past error terms. To build an ARIMA model one essentially use Box-Jenkins methodology (1976), which is an iterative process and involves four stages; Identification, Estimation, Diagnostic Checking and forecasting. As the Box Jenkins (AR, MA, ARMA or ARIMA) models are based on the time series stationary, If underlying series is non-stationary, then first it is converted into a stationary series either by using differencing approach or taking logarithms or regressing the original series against time and by taking the error terms of this regression (Mahesh, 2005). The series stationary was tested by applying the ADF-Augmented Dickey-Fuller (DICKEY & FULLER, 1979) and PP-Phillips-Perron unit root tests (PHILLIPS P., 1988). ADF was performed for the scenario with a constant, without a constant and with a trend (Daniela Spiesov, 2014). If it is needed for the time series to have one differential operation to achieve stationarity, it is a I(1) series. Time series is I(n) in case it is to be differentiated for n times to achieve stationarity. Therefore, ARIMA (p, d, q) models are used for the non-stationary time series, specifically the autoregressive integrated average models, where d is the order of differentiation for the series to become stationary.

Box-Jenkins ARIMA is known as ARIMA (p, d, q) model where p is the number of autoregressive (AR) terms, d is the number of difference taken and q is the number of moving average (MA) terms. ARIMA models always assume the variance of data to be constant. The ARIMA (p, d, q) model can be represented by the following equation:

$$y_t = \emptyset_1 y_{t-1} + \cdots . + \emptyset_p y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} \qquad (1)$$

Where $\varepsilon_t \sim N(0, \sigma_t^2)$, p and q are the number of autoregressive terms and the number of lagged forecast errors, respectively. .

The identification of modeling the conditional mean value is based on the analysis of estimated autocorrelation and partial autocorrelation function (ACF, PACF). These estimations may be strongly inter-correlated, it is therefore recommended not to insist on unambiguous determination of the model order, but to try more models. We must not forget to carry out the verification, which is based on retrospective review of the assumptions imposed on the random errors.

Validation of ARMA (p, q) models is based on minimizing the AIC (Akaik’s information criterion) and BIC (Schwarz’s information criterion) criteria. Given that financial data are very often characterized by high volatility, it is necessary to test the model for ARCH effect, i.e. presence of conditional heteroscedasticity (Mahesh, 2005). Regarding heteroscedasticity it is therefore a situation where the condition of finite and constant variance of random components is violated. If ARCH test indicates that the variance of residuals is non constant, we can use ARCH family models for capturing volatilities of model.

3.2. The ARCH family models

The major assumption behind the least square regression is homoscedasticity i.e constancy of variance. If this condition is violated, the estimates will still be unbiased but they will not be minimum variance estimates. The standard error and confidence intervals calculated in this case become too narrow, giving a false sense of precision. ARCH and related models handle this by modeling volatility itself in the model and thereby correcting the deficiencies of least squares model (AK Dhamija, 2010).

The ARCH Model

A simple strategy is to forecast the conditional variance by an AR (q) process:

$$\hat{\varepsilon}_t^2 = \alpha_0 + \alpha_1 \hat{\varepsilon}_{t-1}^2 + \alpha_2 \hat{\varepsilon}_{t-2}^2 + \cdots + \alpha_q \hat{\varepsilon}_{t-q}^2 + \nu_t \qquad (2)$$

Where ν_t is white noise term. If the amounts of $\alpha_1, \alpha_2, \dots \alpha_q$ are all zero, the estimated variance will be constant and equal to α_0. Otherwise, the conditional variance exists. Hence, the following equation can be used to forecast the conditional variance at the time $t+1$:

$$E_t \hat{\varepsilon}_{t+1}^2 = \alpha_0 + \alpha_1 \hat{\varepsilon}_t^2 + \alpha_2 \hat{\varepsilon}_{t-1}^2 + \cdots + \alpha_q \hat{\varepsilon}_{t+1-q}^2 \qquad (3)$$

Equation (3) is called ARCH model by Engel (1982).

The GARCH Model

Bollerslev (1986) developed the work of Engle in way that the conditional variance be a process of ARMA. Suppose the errors process to be as the following:

$$\varepsilon_t = \nu_t \sqrt{h_t}$$

In a way that $\sigma_\nu^2 = 1$ and

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \hat{\varepsilon}_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \qquad (4)$$

In this condition, one needs to make sure that $\alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0$ and $1 - \sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j > 0$ to see the conditional variance positive. Since ν_t is a white noise, the key point here is that the conditional variance of ε_t is as the following:

$$E_{t-1} \varepsilon_t = h_t$$

So, the ε_t conditional variance complies with an ARMA process like the process (4). Such models are called GARCH (p, q) where q is the number of moving average (MA) terms and p is the number of autoregressive (AR) terms. GARCH model is known as a model of heterocedasticity which means not constant in variance. This model has been used widely in financial and business areas since the data of these areas tend to have variability or highly volatile throughout the time.

The IGARCH Model

Integrated Generalized Autoregressive Conditional Heteroscedasticity (IGARCH) is a restricted version of the GARCH model, where the sum of the persistent parameters sum up to one, and therefore there is a unit root in the GARCH process. The constraints for an IGARCH (p,q) model can be written:

$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j = 0 \quad and \;\; 0 < \beta_j < 1 \tag{5}$$

The Exponential GARCH (EGARCH) Model

A model which accepts the asymmetric effect of the news is the exponential GARCH model (EGARCH). A problem in using the standard model of GARCH is that all the estimated coefficients must be positive. To overcome this problem the exponential GARCH (EGARCH) Model, suggested by Nelson (1991) can be used in which there is no need to observe the condition of non-negativeness for the coefficients:

$$ln(h_t) = \alpha_0 + \alpha_1 \left({}^{\varepsilon_{t-1}}\!/_{h_{t-1}^{0.5}} \right) + \lambda_1 \left| {}^{\varepsilon_{t-1}}\!/_{h_{t-1}^{0.5}} \right| + \beta_1 \ln(h_{t-1}) \tag{6}$$

Three interesting characteristics of the EGARCH model are:

(1) The conditional variance equation has a logarithmic-linear form. Despite the fact that $ln(h_t)$ is large, the amount of h_t cannot be negative. Therefore the coefficients are allowed to be negative.

(2) Instead of using the amount of ε_{t-1}^2, this model uses the standardized amounts ε_{t-1} (ε_{t-1} divide on $h_{t-1}^{0.5}$). Nelson showed that this standardization enables better interpretation of the amount and persistence of the shocks.

(3) The EGARCH receives leverage effect. If ${}^{\varepsilon_{t-1}}\!/_{h_{t-1}^{0.5}}$ is positive, the shock's effect on the conditional variance logarithm will be equal to $\alpha_1 + \lambda_1$. If ${}^{\varepsilon_{t-1}}\!/_{h_{t-1}^{0.5}}$ is negative, the shock's effect on the conditional variance logarithm will be equal to $-\alpha_1 + \lambda_1$.

The Threshold GARCH (TGARCH) Model / GJR Model

A TGARCH (p, q) model as proposed by (Glosten et al., 1993) can also handle leverage effect, but the leverage effect is expressed in a quadratic form while in the case of EGARCH it is expressed in the exponential form. So the GJR-GARCH model is written by:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \hat{\varepsilon}_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} + \sum_{i=1}^{q} \gamma_i \upsilon_{t-i} \hat{\varepsilon}_{t-i}^2 \tag{7}$$

Where

$$\upsilon_{t-i} = \begin{cases} 1, & if\ \varepsilon_{t-i} < 0 \\ 0, & if\ \varepsilon_{t-i} \geq 0 \end{cases} \quad (8)$$

and α_i, γ_i and β_j are non-negative parameters satisfying conditions similar to those of GARCH models. It can be seen that a positive ε_{t-i} contributes $\alpha_i \hat{\varepsilon}_{t-i}^2$ to h_t , wherase a negative ε_{t-i} has a large impact $(\alpha_i + \gamma_i)\hat{\varepsilon}_{t-i}^2$ with $\gamma_i > 0$.

Forecasting Performance Measures

This article uses three different criteria, namely Root Mean Squared Error (RMSE), (Mean Absolute Error) MAE, and Theil Inequality Coefficient (TIC) to compare the performance efficiency of the ARMA and ARMA-GARCH family models in the forecasting of Iran's exchange rate behavior against changes in the U.S. Dollar (IRR/USD). That model with a smaller amount would be the considered as a better and more appropriate model.

1. Root Mean Squared Error (RMSE): Root Mean Square Error (RMSE) measures the difference between the true values and estimated values, and accumulates all these difference together as a standard for the predictive ability of a model. The criterion is the smaller value of the RMSE, the better the predicting ability of the model. It is defined as follows:

$$RMSE = \sqrt{\frac{\sum_{t=T+1}^{T+k} (\hat{y}_t - y_t)^2}{n}} \quad (9)$$

2. Mean Absolute Error (MAE): It takes into consideration the average of the absolute value of the residuals. It is:

$$MAE = \sum_{t=T+1}^{T+k} \left| \frac{\hat{y}_t - y_t}{n} \right| \quad (10)$$

3. Theil Inequality Coefficient (TIC): The Theil inequality coefficient always lies between zero and one, where zero indicates a perfect fit.

$$TIC = \frac{\sqrt{\sum_{t=T+1}^{T+k} (\hat{y}_t - y_t)^2}}{\sqrt{\sum_{t=T+1}^{T+k} \hat{y}_t{}^2} + \sqrt{\sum_{t=T+1}^{T+k} y_t{}^2}} \quad (11)$$

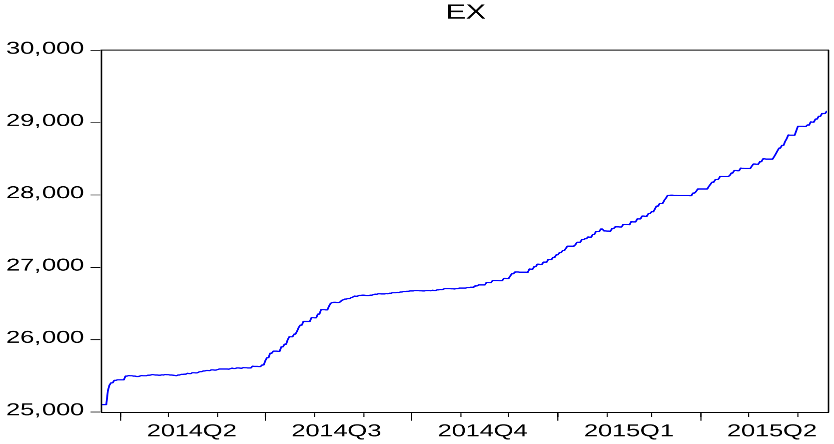
Where y_t is observed values, $\hat{y}_t$ is the predicted values at time t and n is the number of forecasts.

4. Empirical results

4.1 Data and Stationary Examination of Variable

Daily data of Iran's exchange rate against the U.S. Dollar for period 20-3-2014 to 20-6-2015have been derived from Central Bank of Iran reports. Figure 1 shows the changes of Iran's daily exchange rate for this period.

Figure 1.Daily data of exchange rate for the period of 20-3-2014 to 20-6-2015



Since the basis of Box-Jenkins models' forecasting is the stationary of the series in question, so we use of Augmented Dickey–Fuller (ADF) test and Phillips-Perron (PP) test on exchange rate data. Table 1 summarized the unit root tests for exchange rate series. The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests were used to test the null hypothesis of a unit root against the alternative hypothesis of stationarity. According to the Table 1 the results of ADF and PP tests show that the exchange rate series is non stationary, because the statistic value for both ADF and PP tests are greater than their corresponding critical values.

Table 1. ADF and Phillip-Perron test on exchange rate series.

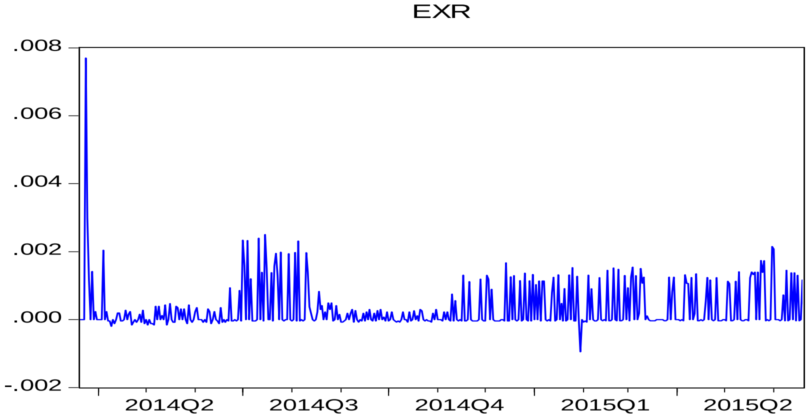
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		0.963483	0.9963
Test critical values:	1% level	-3.445445	
	5% level	-2.868089	
	10% level	-2.570323	
		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		0.671980	0.9915
Test critical values:	1% level	-3.445445	
	5% level	-2.868089	
	10% level	-2.570323	

To transform the non stationary exchange rate series, we calculate the exchange rate returns as:

$$EXR = \log(EX_t) - \log(EX_{t-1}) = \log(EX_t / EX_{t-1}) \qquad (12)$$

The time series plot of the transformed data that is named exchange rate returns is shown in Figure 2. This plot shows that the mean of the series is now about constant. Hence, we can assume that the series is stationary. The variance is high that clearly exhibit volatility clustering, which allows us to carry on further to apply the ARCH family models.

Figure 2.Daily data of exchange rate returns (EXR) for the period of 20-3-2014 to 20-6-2015



In Table 2 the results of ADF test and PP test show that the exchange rate returns series is stationary.

Table 2. ADF and Phillip-Perron test on exchange rate return series.

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-12.19427	0.0000
Test critical values:	1% level	-3.445445	
	5% level	-2.868089	
	10% level	-2.570323	
		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-18.94206	0.0000
Test critical values:	1% level	-3.445445	
	5% level	-2.868089	
	10% level	-2.570323	

To assess the distributional properties of the exchange rate return data, various descriptive statistics are reported in Table 3.

Table 3. Summary statistics of Iran's Exchange Rate Returns (IRR/USD).

Mean	Std. Dev.	Skewness	Kurtosis	Jarque Bera	Prob.*
0.00033	0.00069	3.72	31.30	16203.57	0.00

Table 3 shows that the mean of exchange rate returns is close to zero and the sample kurtosis for it is well above the normal value of 3. There is also evidence of positive skewness, with long right tail indicating that exchange rate has non symmetric returns. Jarque-Bera value shows that exchange rate returns distribution is leptokurtic and depart significantly from Gaussian distribution. Therefore, for capturing of volatilities in time series of returns, we will use the autoregressive conditional heteroscedasticity (ARCH) family models.

4.2. Model Estimation and Forecasting using the Box-Jenkins Method

In this section we tried to build univariate model to forecast exchange rate of Iran in terms of USD using Box-Jenkins Methodology of building ARIMA model. In order to find the most optimal lags, different AR and MA lags were tested. Autocorrelation and partial autocorrelation functions of residuals are also used. Information criteria of Akaike and Schwarz were also employed for identifying the best model. The most appropriate obtained model among different models is the following ARIMA ((2,4,11), (4)) type that is an adequate choice:

$$\mathrm{EXR_t} = 0.0003 + 0.08\mathrm{EXR_{t-2}} - 0.52\mathrm{EXR_{t-4}} + 0.14\mathrm{EXR_{t-11}} + 0.768\varepsilon_{t-4} + \varepsilon_t \quad (13)$$

Where EXR represents the exchange rate returns. The p-values of the t-statistic of the estimated coefficients showed that all of them are highly significant. No evidence autocorrelation was found in this model's residuals (using the LM test) and D.W for this model is 2.001, Akaike info criterion (AIC) and Schwarz criterion (SBIC) are -11.955 and -11.905, respectively. This ARIMA model is used to forecast the exchange rate returns for the period (20-4-2015 to 20-6-2015). The RMSE, MAE and TIC values are 0.000833, 0.000667 and 0.608215, respectively.

As it is shown in Table 4, the serial correlation LM test (Breusch-Godfrey test) indicates that the estimated residuals are not autocorrelation. Hence, there is no need to search out for another ARIMA model.

Table 4. Serial Correlation LM Test on ARMA residuals.

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	1.251633	Prob. F(2,389)	0.2872
Obs*R-squared	2.524930	Prob. Chi-Square(2)	0.2830

The next step is to test whether the estimated errors are heteroscedastic or not. For this purpose, we test the presence of 'ARCH effect' in the residuals by using the Lagrange Multiplier (LM) test for exchange rate returns series as suggested by Engle. The results of Lagrange Multiplier test are presented in Table 5. The p-value indicates that there is evidence of remaining ARCH effect. So, we reject the null hypothesis of absence of ARCH effect even at 5% level of significance. Hence, in next section for capturing volatilities in exchange rate returns series we will use GARCH family of models.

Table 5. Heteroskedasticity Test: ARCH for ARMA Model.

Heteroskedasticity Test: ARCH

F-statistic	5.219649	Prob. F(2,391)	0.0058
Obs*R-squared	10.24584	Prob. Chi-Square(2)	0.0060

4.3. Estimation and forecasting Based on the various GARCH models.

Exchange rate series is taken from 20-03-2014 to 20-06-2015 and various GARCH models are fitted to the exchange rate returns from 20-03-2014 to 19-04-2015. The joint estimation of mean and variance equations using 'Eviews' software is shown below for various GARCH models.

ARIMA –ARCH Model

A joint estimation of the ARIMA-ARCH(2) model gives:

Mean equation:

$$\mathrm{EXR_t} = 0.00028 + 0.24\mathrm{EXR_{t-7}} + 0.10\mathrm{EXR_{t-11}} + 0.06\mathrm{EXR_{t-12}} + 0.08\varepsilon_{t-4} + \varepsilon_t \quad (14)$$

Variance equation:

$$h_t = 0.00000009 + 0.149\varepsilon_{t-1}^2 + 0.841\varepsilon_{t-2}^2 \quad (15)$$

In Equation (15) h_t is the conditional variance. The amount of p_value for parameters of mean equation and variance equation are 0.00. So, all of the coefficients are highly significant. Akaike info criterion (AIC) and Schwarz criterion (SBIC) are -12.2917 and -12.2113, respectively. The RMSE, MAE and TIC values for forecasted data of this model are 0.000687, 0.000575 and 0.542451, respectively. Now, we check whether the ARIMA – ARCH has adequately captured the persistence in volatility and there is no ARCH effect left in the residuals from the selected models. The ARCH LM test is conducted for this purpose. The results of LM test given in Table 6 indicate that the residuals do not show any ARCH effect. Hence, ARIMA – ARCH is found to be reasonable to remove the persistence in volatility.

Table 6. Heteroskedasticity Test: ARCH for ARMA-ARCH Model.

Heteroskedasticity Test: ARCH

F-statistic	0.617188	Prob. F(1,393)	0.4326
Obs*R-squared	0.619356	Prob. Chi-Square(1)	0.4313

ARIMA –IGARCH Model

Equation (15) shows that sum of coefficients ARCH terms is closely to one. So, for promotion the model, we apply an IGARCH model for capturing volatilities of returns series. A joint estimation of the ARIMA-IGARCH (1,1) model gives:

Mean equation:

$$EXR_t = 0.00022 + 0.19EXR_{t-7} + 0.14EXR_{t-11} + 0.13EXR_{t-12} + 0.10\varepsilon_{t-4} + \varepsilon_t \quad (16)$$

Variance equation:

$$h_t = 0.210021\varepsilon_{t-1}^2 + 0.789979h_{t-1} \quad (17)$$

The amount of p_value for parameters of mean equation and variance equation are 0.00. So, all of the coefficients are highly significant. Akaike info criterion (AIC) and Schwarz criterion (SBIC) are -12.4391 and -12.3788, respectively. The RMSE, MAE and TIC values for forecasted data of this model are 0.000707, 0.000583 and 0.559248, respectively. The ARCH LM test indicates that the residuals do not show any ARCH effect.

As we mentioned skewness and kurtosis represent the nature of departure from Normality and positive or negative skewness indicate asymmetry in the series. According to the table (3), skewness of 3.72 for exchange rate returns series indicates positively skewed due to the leverage effect with long right tail and a deviation from Normality. In the following the Glosten, Jagannathan and Runkle (GJR) and EGARCH models are used to test the leverage effect that successfully captures the asymmetry.

ARIMA –GJR Model

The results on mean equation and asymmetric conditional variance for exchange rate returns series by using Glosten, Jagannathan and Runkle (ARIMA-GJR-GARCH (1,1)) Model are reported as follow:

Mean equation:

$$EXR_t = 0.0035 + 0.0478EXR_{t-7} + 0.074EXR_{t-11} - 0.066\varepsilon_{t-4} - 0.108\varepsilon_{t-12} + 0.000205\log(h_t) + \varepsilon_t \quad (18)$$

Variance equation:

$$h_t = 0.000000011 + 0.181\varepsilon_{t-1}^2 + 0.594h_{t-1} + 0.304\varepsilon_{t-1}^2 * (\varepsilon_{t-1} < 0) \quad (19)$$

The results are showed that all of the coefficients are statistically significant at 5% level. Akaike info criterion (AIC) and Schwarz info criterion (SBIC) are -12.620 and -12.520, respectively. The RMSE, MAE and TIC values for forecasted data of this model are 0.000720, 0.000675 and 0.502462, respectively. The ARCH LM test indicated that the ARIMA –GJR has adequately captured the persistence in volatility and there is no ARCH effect left in the residuals from the selected models. The mean equation consists of $\log(h_t)$ term, so this model named ARIMA-GJR-GARCH_MEAN also.

ARIMA – EGARCH Model

ARMA – EGARCH (2,1) model for exchange rate returns series is estimated as:

Mean equation:

$$\mathrm{EXR_t} = 0.00037 + 0.209\mathrm{EXR_{t-2}} + 0.199\mathrm{EXR_{t-7}} + 0.093\varepsilon_{t-12} + \varepsilon_t \qquad (20)$$

Variance equation:

$$\log(h_t) = -24.43 + 0.639\frac{|\varepsilon_{t-1}|}{\sqrt{\log(h_{t-1})}} + 0.697\frac{|\varepsilon_{t-2}|}{\sqrt{\log(h_{t-2})}} - 0.177\frac{\varepsilon_{t-1}}{\sqrt{\log(h_{t-1})}} - 0.557\log(h_{t-1}) \qquad (21)$$

The results are showed that all of the coefficients are statistically significant at 5% level. Akaike info criterion (AIC) and Schwarz criterion (SBIC) for this model are -12.1041 and -12.0137, respectively. The RMSE, MAE and TIC values for forecasted data of this model are 0.000661, 0.000569 and 0.500069, respectively. The ARCH LM test indicated that the ARIMA –EGARCH has adequately captured the persistence in volatility and there is no ARCH effect left in the residuals from the selected models.

4.4 COMPARATIVE ANALYSIS

In order to assess the validity of forecasting the exchange rate returns through the models presented in this paper, the RMSE, MAE and TIC criteria of these models are compared with each other. Table (7) presents RMSE, MAE, TIC for estimated models in this paper.

Table 7. Comparison of test statistics for ARMA and family of GARCH models

MODEL	RMSE	MAE	TIC
ARIMA	0.000833	0.000667	0.608215
ARIMA-ARCH	0.000687	0.000575	0.542451
ARIMA-IGARCH	0.000707	0.000583	0.559248
ARIMA-GJR	0.000720	0.000675	0.502462
ARIMA-EGARCH	0.000661*	0.000569*	0.500069*

(*) minimum values to criterion.

According to the achieved results of Table 7, the ARIMA-EGARCH model has the best value for RMSE, MAE and TIC criteria equal to 0.000661, 0.000569 and 0.500069, respectively. So, the comparison of the forecasting performance through the RMSE, MAE and TIC criteria indicate that the best model is ARIMA-EGARCH .Therefore, ARIMA-EGARCH model captures the volatility and leverage effect in the exchange rate returns and its forecast performance is more than other models. Although this selected model has not the lowest values of diagnostic checking (such as AIC and SBCI), this result agrees with Mujumdar and Nagesh Kumar (1990) that best model for representation the data and best model for forecasting are often not the same.

5. Conclusion

The time series forecasting plays a central role in risk management, portfolio selection, asset valuations, option pricing, and hedging strategies in modern Finance. This paper focuses on building a model for the exchange rate of Iran using time series methodology. Daily data of exchange rate RRI/USD for the period ranging from 20 March 2014 to 20 June 2015 are used for this purpose. First of all, the stationary of the exchange rate series is examined using unit root test such as ADF and PP tests which showed the series as non stationary. Hence, to make the exchange rate series stationary, the exchange rates are transformed to exchange rate returns. In order to find the most optimal lags, different AR and MA lags were tested using the Box-Jenkins Method. The most appropriate obtained model among different models using AIC and BIC is the ARIMA ((2,4,11),(4)). As the financial time series like exchange rate returns may possess volatility, an attempt is made to model this volatility using ARCH/GARCH family models. To capture the volatility, ARIMA ((7,11,12),(4))–GARCH(2,0) model is used. The sum of coefficients of this model was very close to one. So, we estimated an ARIMA ((7,11,12),(4)) -IGARCH(1,1) model. Because of positive skewness and asymmetries of returns series, an ARIMA((7,11),(4,12))–GJR(1,1) and an ARIMA((7,2),(12)) –EGARCH(2,1) model are estimated so that capture leverage effect in returns series.

Finally, the forecast performance is measured using different measures like RMSE, MAE, and TIC. The ARIMA-EGARCH is found to be the best model with the lowest RMSE, MAE, and TIC. This model captures the volatility and leverage effect in the exchange rate returns and provides a model with fairly good forecasting performance.

In further research, the above models that applied in IRR/USD exchange rate forecasting could easily be applied in other exchange rates also.

References

1- Abdullah, S. Karaman and Tayfur, Altok (2004), An Experimental Study on Forecasting Using Tes Processes. Proceedings of the winter simulation conference, R. G. Ingalls, M. D. Rossetti, J. S. Smith, and B. A. Peters, eds., PP.437-442.

2- Ajoy, K.Palit and Dobrivoje, Popovic (2005), Computational Intelligence in Time Series Forecasting – Theory and Engineering Applications. Springer–Verlag London Limited.

3- AK Dhamija and VK Bhalla (2010), Financial Time Series Forecasting: Comparison of Various Arch Models, Global Journal of Finance and Management, Vol.2, No.1, pp. 159-172.

4- Bilson, J.F.O (1978), Rational expectations and the exchange rate, in J.A. Frankel and H.G. Johnson (eds.), The Economics of Exchange Rate (Reading, Mass, Addison-Wesley), pp 75-96.

5- Bollerslev, Tim. (1986), Generalized Autoregressive Conditional Heteroskedasticity. Journal of Econometrics 31, 307-27.

6- Box, G.E.P., Jenkins, G.M (1976), Time Series Analysis – Forecasting and Control. Holden-Day Inc., San Francisco.

7- Carrol R. and Kearvey C (2009), GARCH Modeling of stock market volatility in Gregoriou G. (ed), Stock Market Volatility, CRC Finance Series, CRC Presss, 71-90.

8- Chatayan Wiphatthananthakul, and Songsak Sriboonchitta (2010), The Comparison among ARMA-GARCH, -EGARCH, -GJR, and -PGARCH models on Thailand Volatility Index, The Thailand Econometrics Society, Vol. 2, No. 2, 140 – 148.

9- Daniela Spiesov (2014), The Prediction of Exchange Rates with the Use of Auto-Regressive Integrated Moving-Average Models, ACTA UNIVERSITATIS DANUBIUS, AUDOE, Vol. 10, no. 5, pp. 28-38.

10- Dickey, D. A. & Fuller, W. (1979), Distribution of estimators for autoregressive time series with a unit root. J. Am. Stat. Assoc. 74, pp. 427–431.

11- Dornbusch, R.(1976), Expectations and exchange rate dyanamics, Journal of Political Economy, 84, pp 1161-1176.

12- Dunis, C. & Huang, X. (2002), Forecasting and trading currency volatility: An application of recurrent neural regression and model combination. England, Liverpool.

13- Enders Walter,(2003), Applied Econometric Time Series; 2nd (Second) edition, Publisher: Wiley, John & Sons.

14- Engle, R.F. (1982). Autoregressive Conditional Heteroscedasticity with estimates of variance of United Kingdom inflation. Econometrica, 50(4), 987-1007.

15- Fat, C.M. & Dezci, E. (2011), Exchange-rates forecasting: exponential smoothing techniques and Arima models. [online] Faculty of Economics and

Business Administration, Department of Finance, “Babes-Bolyai” University, Cluj-Napoca, Romania, Retrieved from:ttp://steconomiceuoradea.ro/anale/volume/2011/n1/046.pdf.
16- Frankel, J. On the mark. (1979), A theory of floating exchange rates based on real interest differentials, American Economic Review, 69, pp. 610-622.
17- Frankel, J. (1979), On the mark: A theory of floating exchange rates based on real interest differentials, American Economic Review, 69, pp. 610-622.
18- Glosten, L.,Jagannathan, R. and Runke, D. (1993), Relationship between the expected value and the volatility of the nominal excess return on stocks. Journal of Finance 48, 1779– 1801.
19- Green, W.H (2000), Econometric Analysis; Macmillan,(4th edition), New York University.
20- Gujarati, DamodarN. (2006), Essentials of Econometrics. 3th Edition, with Eviews 4,1. McGraw-Hill Higher Education Publishing.
21- Hamilton, James, and Raul Susmel. (1994), Autoregressive Conditional Heteroskedasticity and Changes in Regime. Journal of Econometrics 64,307-33.
22- Herreraa, L.J., Pomaresa, H., Marquezb, L., Pasadas, M., (2008), Soft-computing techniques and ARMA model for time series prediction. Neurocomputing 71, pp519–537.
23- John Faust, John H. Rogers, Jonathan H. Wright (2002), Exchange rate forecasting: the errors we’ve really made, Journal of International Economics, 60,pp 35-59.
24- Lindsay I. Hogan (1985), A comparison of alternative exchange rate forecasting models, Bureau of agricultural Economics, Canberra ACT 2601.
25- Lutz Lillian and Mark P. Taylor (2003), Why it is so difficult to beat random walk forecast of exchange rates?, Journal of International Economics,60,pp85-107.
26- Mahesh Kumar Tambi (2005), FORECASTING EXCHANGE RATE: A Uni-variate out of sample Approach (Box-Jenkins Methodology), in The IUP Journal of Bank Management, http://econwpa.repec.org/eps/if/papers/0506/0506005.pdf
27- Meese, R. A. and K. Rogoff, (1983), Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample? Journal of International Economics, pp.3-24.
28- Meese, R. A. and K. Rogoff, (1983), The Out-of-Sample Failure of Empirical Exchange Rate Models: Sampling Error or Misspecification? in Exchange Rate and Interantional Macroecomics, J. Frenkel ed., University of Chicago Press, Chicago, pp.67-112.
29- Minakhi Rout, Babita Majhi, Ritanjali Majhi and Ganapati Panda, (2014), Forecasting of currency exchange rates using an adaptive ARMA model with

differential evolution based training , Journal of King Saud University – Computer and Information Sciences 26, 7–18
30- M.K. Newaz. (2008), Comparing the performance of time series models for forcasting Exchange Rate, , BRAC University Journal, vol. V, no. 2, pp. 55-65
31- Milton Abdul Thorlie, Lixin Song, Xiaoguang Wang, Muhammad Amin. (2014), Modelling Exchange Rate Volatility Using Asymmetric GARCH Models (Evidence from Sierra Leone), International Journal of Science and Research (IJSR), Volume 3 Issue 11, PP 1206-1214.
32- Moshiri Saeed and Seifi Forough (2008), Nonlinearity in Exchange Rates and Forecasting, Iranian Economic Review, Vol.13, No.21.
33- Mujumdar,P.P., and Nagesh Kumar,D. (1990), Stochastic models of streamflow Somecase studies.Hydrol.Sci.J.35,pp.395-410.
34- Nelson, D.B. (1991). Conditional heteroscedasticity in asset returns: A new approach. Econometrica, 59(2), 347-370.
35- Nwankwo Steve C (2014), Autoregressive Integrated Moving Average (ARIMA) Model for Exchange Rate (Naira to Dollar), Academic Journal of Interdisciplinary Studies, MCSER Publishing, Rome-Italy, Vol 3 .No 4, pp 429-433
36- Phillips, P. C. B. & Perron, P. (1988), Testing for a unit root in time series regression. Biometrika 75, pp. 335–346.
37- Shahla Ramzan, Shumila Ramzan, Faisal Maqbool Zahid (2012), Modeling and Forecasting Exchange Rate Dynamics in Pakistan using Arch Family of Models, Electronic Journal of Applied Statistical Analysis, Vol. 5, Issue 1, 15 – 29.
38- S.R. Yaziz, N.A. Azizan, R. Zakaria, M.H. Ahmad(2013), The performance of hybrid ARIMA-GARCH modeling in forecasting gold price, 20th International Congress on Modelling and Simulation, Adelaide, Australia, 1–6 December 2013,www.mssanz.org.au/modsim2013
39- Wang, W., Van Gelder, P. H. A. J. M., Vrijing, J. K. & Ma, J. (2005), Testing and modeling autoregressive conditional heteroskedasticity of streamflow processes. Nonlinear Processes in Geophysics, 12, 55-66.
40- Weisang, G. &Yukika, A. (2008), Vagaries of the Euro: an Introduction to ARIMA Modeling, [online] Bentley College, USA, Retrieved from http://www.bentley.edu/centers/sites/www.bentley.edu.centers/files/csbigs/weisang.pdf.